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Hypersonic, Turbulent Viscous Interaction past an Expansion Corner

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Turbulent, viscous interaction theory is used to understand the hypersonic flow past an expansion corner. By assuming a pressure law, the boundary-layer properties of the flow are obtained through simultaneous solution of a displacement thickness relationship and a coupling equation relating the effects of incidence and displacement thickness to the effective body shape. The pressure law is obtained through examination of available experimental data. The form of the pressure law is found to depend on the incidence. The pressure decay is dependent on both viscous and incidence effects. For weak incidence, viscous effects are important throughout the interaction, and they produce a gentle pressure decay. For strong incidence, viscous effects are important only near the front of the corner. In this latter case, the pressure decays rapidly followed by a long asymptotic downstream region; the overall pressure distribution then appears more similar to the inviscid case.

Nomenclature

A	$= \gamma$
B	$= \gamma(\gamma + 1)/2$
C_∞	= constant of proportionality in linear viscosity-temperature relationship
K	= hypersonic similarity parameter, $M_\infty \alpha$
K_v	= modified hypersonic similarity parameter, $M_\infty \, dy_e(x)/dx$
M	= Mach number
$P(x)$	= nondimensional pressure, $p_e(x)/p_\infty$
p	= pressure
Re	= unit Reynolds number
T	= temperature
U	= velocity
(x, y)	= physical coordinates along and normal to the incoming flow
α	= expansion corner angle
γ	= ratio of specific heats
δ	= boundary-layer thickness
δ^*	= boundary-layer displacement thickness
ξ	= dummy variable of integration
ρ	= density
$\bar{\chi}_w$	= weak turbulent, viscous interaction parameter, $(M_\infty^9 C_\infty / Re_x)^{1/5}$

Subscripts

e	= edge condition or effective
w	= wall condition
∞	= incoming freestream condition
0	= stagnation condition

I. Introduction

IN recent years, increasing interest in the development of flight vehicles of extremely high velocities has motivated a large number of research studies. At hypersonic speeds where the flight velocity is far greater than the speed of sound, the characteristics of the flow can be drastically different than those at supersonic speeds. These hypersonic features have brought a number of fluid dynamic problems into prominence, one of which is a class of viscous interactions due to the presence of thick boundary layers.

Viscous interaction was first identified in pressure measurements near the leading edge of a sharp wedge by Becker.[1] He found that the actual pressure along the wedge is considerably higher than the inviscid pressure obtained from oblique shock theory. Viscous interaction or pressure interaction[2] can be viewed as the mutual interaction between the external inviscid flowfield and the boundary layer around a body. For example, a laminar boundary layer grows on a flat plate as $M_\infty^2/\sqrt{Re_\delta}$ (Ref. 2). Therefore, relatively speaking, in subsonic and most supersonic flows where the Reynolds number is much larger than the square of the Mach number, boundary-layer growth in changing the effective body shape or the actual pressure distribution can be ignored in general. In hypersonic flight, this is no longer so, and viscous interaction due to the presence of a comparatively thick boundary layer can become significant. As a cautionary note, however, viscous interactions are not always the dominant feature in a hypersonic flow.[2]

The rapid thickening of the boundary-layer in hypersonic flow distorts the external inviscid flowfield severely, which in turn modifies the boundary-layer growth. A mutual viscous-inviscid interaction is set up. For certain flows, an analytical procedure can be implemented by including the boundary-layer displacement thickness into an effective body shape and calculating the corresponding pressure distribution.

There are several examples of viscous interactions which are of interest in both laminar and turbulent boundary layers. Some of those that have been studied analytically include hypersonic flow near the leading edge of a sharp flat plate and unseparated shock boundary layer interactions in turbulent flow. In such analytical studies, the strong pressure gradient is usually induced by changes of surface slope, e.g., at the hinge line of a corner.

The classical, viscous interaction theory[2] was first formulated by Cheng et al.[3] for laminar flows. Barnes and Tang[4] later identified the strong and weak viscous interaction parameters in turbulent flow whereas Stollery and Bates[5] extended Cheng's laminar theory for turbulent interactions. They compared their theory with experimental data obtained by Coleman and Stollery[6] and Elfstrom.[7] The present paper extends the turbulent, viscous interaction theory[5] to the case of expansion corners (Fig. 1) in a straightforward way. Satisfactory results are obtained in comparisons with previous experiments.[8–10]

II. Brief Review of Turbulent, Viscous Interaction Theory

Hypersonic flow past bodies with sharp leading edges can be represented by three equations.[5] These equations are a boundary-

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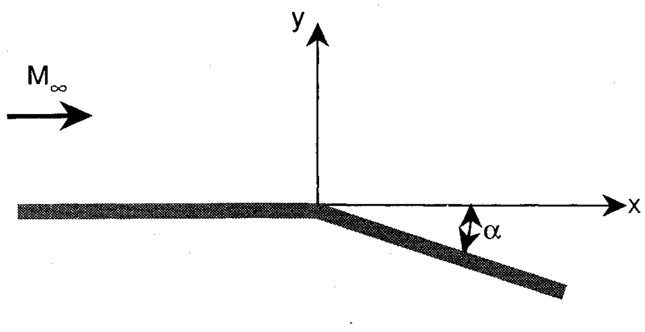

Fig. 1 Schematic of expansion corner.

layer equation, an inviscid flow equation and a coupling equation, namely,

$$\delta^* = f_1(P) \tag{1}$$

$$P = f_2(y_e) \tag{2}$$

$$y_e = f_3(\delta^*) \tag{3}$$

where $P = p_e(x)/p_\infty$ is an initially unknown, nondimensional pressure distribution. The pressure is assumed constant across the boundary layer so that $p_e = p_w$, but due to viscous interaction the pressure at the outer edge of the boundary layer p_e is not the same as the incoming freestream pressure p_∞. This very general formulation of the problem suggests that a complete solution to the flow past a given shape can be obtained once the boundary-layer growth, the external pressure distribution, and the effective body shape are known. Although many different choices for these equations are possible, simple forms of these equations are employed to ensure tractability. These choices for Eqs. (1–3) are discussed next. Of these, Eq. (2) produces the greatest difficulty and is discussed last.

A. Displacement Thickness

A general expression for displacement thickness for turbulent flows was obtained by Stollery and Bates[5] based on the momentum-integral relation, namely,

$$\frac{M_\infty \delta^*(x)}{x} = 0.051\frac{1+1.3T_w/T_0}{(1+2.5T_w/T_0)^{3/5}}\bar{\chi}_w \times \left[\left(\int_0^x P^\eta \, \mathrm{d}\xi\right)\Big/x\right]^{4/5} \Big/ P^\kappa \tag{4}$$

where

$$\eta = (5.75 - 1.625T_w/T_0)/7 \tag{5}$$

$$\kappa = (6 - 1.3T_w/T_0)/7 \tag{6}$$

and $\bar{\chi}_w$ is the weak turbulent, viscous interaction parameter; $P(x)$ is an initially unknown pressure distribution; and ξ is the dummy variable of integration in x. Equation (4) describes the growth of the boundary-layer displacement thickness in an unknown pressure gradient for any given Mach number and wall temperature ratio. It is assumed to be valid in expansive flows so long as the flow is not relaminarizing.

B. Effective Body Shape

Once $\delta^*(x)$ is known, it is added to the actual body shape to produce an effective body shape $y_w(x)$, i.e.,

$$y_e(x) = y_w(x) + \delta^*(x) \tag{7}$$

Equation (7) can be differentiated with respect to x to yield

$$\frac{\mathrm{d}y_e}{\mathrm{d}x} = \frac{\mathrm{d}y_w}{\mathrm{d}x} + \frac{\mathrm{d}\delta^*}{\mathrm{d}x} \tag{8}$$

from which $K_v = M_\infty\, \mathrm{d}y_e/\mathrm{d}x$ can be evaluated. It may be noted here that Eq. (8) is an approximate relationship. The exact relationship for this coupling equation[11,12] is

$$\frac{\mathrm{d}y_e}{\mathrm{d}x} = \frac{\mathrm{d}y_w}{\mathrm{d}x} + \frac{\mathrm{d}\delta^*}{\mathrm{d}x} - (\delta - \delta^*)\frac{\mathrm{d}}{\mathrm{d}x}\ln(\rho_e U_e) \tag{9}$$

For hypersonic boundary layers, as the Mach number tends to infinity, the displacement thickness approaches the boundary-layer thickness. Therefore, at high Mach numbers, the last term of Eq. (9) is small and can be neglected.

C. Pressure Law for Expansion Corners

However, at this stage, a pressure relationship is still to be formulated. The importance of the choice of the pressure law in solving hypersonic, viscous interaction problems including the effect of incidence was highlighted by Stollery[13] and Mohammadian.[14] Various choices for the pressure law exist. The first to be studied here is a modified form of the tangent-wedge rule given by[13]

$$P = 1 + \gamma K_v^2\left(\frac{\gamma+1}{4} - \left\{\left(\frac{\gamma+1}{4}\right)^2 + \frac{1}{K_v^2}\right\}^{1/2}\right) \tag{10}$$

Although at first glance Eq. (10) appears to be suitable for expansion corners, the presence of K_v imposes a condition that results in an underprediction of the pressure values, as will be seen shortly.

The second available choice for the pressure law is

$$\frac{p_2}{p_1} = \left(1 - \frac{\gamma-1}{2}K\right)^{2\gamma/(\gamma-1)} \tag{11}$$

where $K = M_\infty \alpha$ is the hypersonic similarity parameter. Note that K in Eq. (11) is not the same as $K_v = M_\infty\, \mathrm{d}y_e/\mathrm{d}x$ and, in fact, the inviscid parameter K is inappropriate for obtaining solutions to a viscous interaction problem as will be explained later.

In the present study, the following pressure laws are suggested, namely,

$$P = 1 + AK_v + BK_v^2 + \bar{\chi}_w^{-0.4}K \tag{12}$$

for $K \leq 1$, and

$$P = 1 + AK_v + BK_v^2 + \bar{\chi}_w^{0.4}K \tag{13}$$

for $K > 1$. Note that both Eqs. (12) and (13) give $P = 1$ as required to obtain the flat plate, zero incidence solution when both K and K_v are zero. The coefficients A and B are functions of the specific heat ratio, see the Nomenclature. Specifically, the first three terms on the right-hand side of Eqs. (12) and (13) incorporate the weak and strong approximations to the tangent-wedge rule as given by Stollery and Bates.[5] The last term in these equations describes the joint effects of viscous interaction and incidence through the weak viscous interaction parameter $\bar{\chi}_w$ and the parameter K. The power of $\bar{\chi}_w$, −0.4 and 0.4 in the respective equations, is empirically determined through comparisons with experimental data,[8–10] and such comparisons will be discussed later. It is found that for strong expansions where $K > 1$, the negative power of $\bar{\chi}_w$ in Eq. (12) is inadequate to produce the attenuation of pressure downstream and, therefore, a positive power is required. In other words, for cases where the Mach number and the incidence are large, i.e., $K > 1$, the necessary correction in Eq. (13) is considerably larger compared to cases where $K \leq 1$ in Eq. (12). It may also be noted that for $K = 1$, a nonphysical discontinuity will develop if both Eqs. (12) and (13) are assumed to be valid.

III. Application and Discussion

The characteristics of hypersonic, turbulent flow past an expansion corner can be predicted by simultaneous solution of the appropriate pressure law, Eq. (12) or (13), the displacement thickness relationship Eq. (4), and the coupling equation, Eq. (8). The solution is started once the initially unknown pressure P in Eq. (4) is specified. Two different choices for the initially unknown pressure P have been considered in this study, namely, a power law assumption

$$P = ax^m \tag{14}$$

and an exponential pressure change[15,16]

$$P = b \exp(sx) \tag{15}$$

By matching to the upstream, flat plate solution at the hinge line, unique sets of constants (a, m) or (s, b) can be obtained.[5,17] Equations (14) and (15) assume no upstream influence. The lack of an upstream influence is a feature that is observed in turbulent, expansion corner flows[9] but not in laminar, expansion corner flows.[18] Numerical solutions can be initiated by substituting Eq. (14) or Eq. (15) into Eq. (4). Now the solution to the problem can be generated

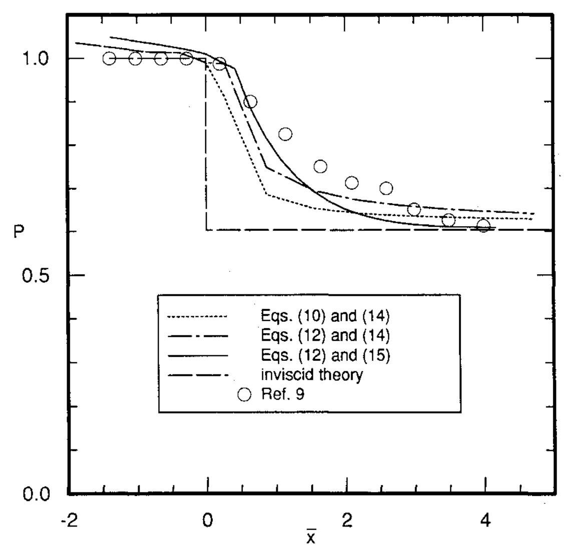

Fig. 2 Comparison with Lu and Chung's data,[9] 2.5-deg expansion corner at Mach 8.

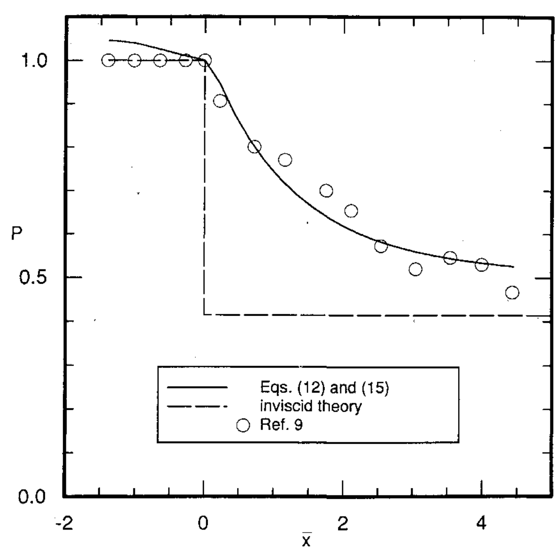

Fig. 3 Comparison with Lu and Chung's data,[9] 4.25-deg expansion corner at Mach 8.

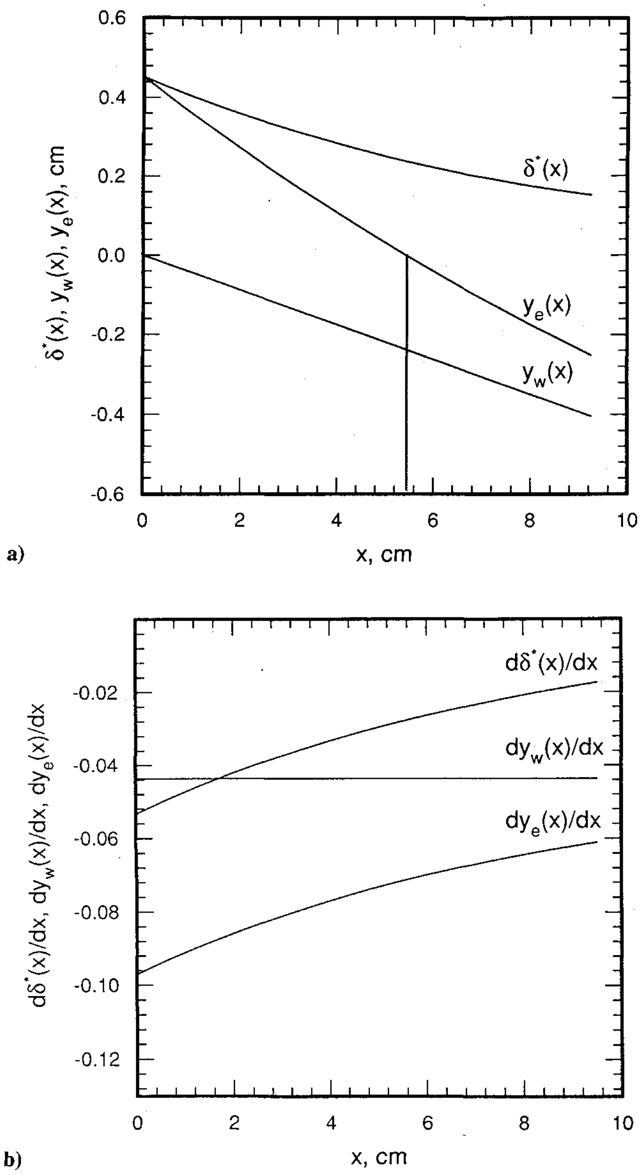

Fig. 4 The relative effects of displacement thickness and incidence for 2.5-deg expansion in a Mach 8 flow: a) geometric parameters and b) slope of geometric parameters.

by simultaneously solving Eqs. (4), (8), and a pressure law such as Eqs. (10), (12), or (13), depending on the value of K (Ref. 17).

Figure 2 compares results using different pressure laws with experimental data[9] for hypersonic, turbulent viscous flow past a 2.5-deg expansion corner. (The initial conditions for the present viscous calculations are identical to those of Lu and Chung[9] where T_w = 290 K, T_0 = 820 K, $M_\infty = 8$, $Re = 10.2 \times 10^6$/m.) The inviscid pressure distribution is also displayed. The nondimensional pressure is plotted against $\bar{x} = x/\delta_0$, where the undisturbed boundary-layer thickness at the corner location $\delta_0 = 1.35$ cm. This value of δ_0 is used to match the experiment with the theory. It can be seen that if the initially unknown pressure P in Eq. (4) assumes the power law form of Eq. (14), neither one of the pressure laws given by Eqs. (10) or (12) predict the pressure distribution accurately. However, once P takes the exponential form of Eq. (15) and is used in conjunction with Eq. (12) the pressure distribution is in good agreement with experimental data. Note that Eq. (12) is used since $K < 1$. It can be deduced that the exponential assumption is the appropriate choice for this type of calculation as observed previously.[15,16] Therefore, for the remaining cases this assumption is used. In Fig. 2, the lines depicting the viscous theory show discontinuities arising from numerical integration. Note that the interaction theory correctly obtains the asymptotic behavior toward the

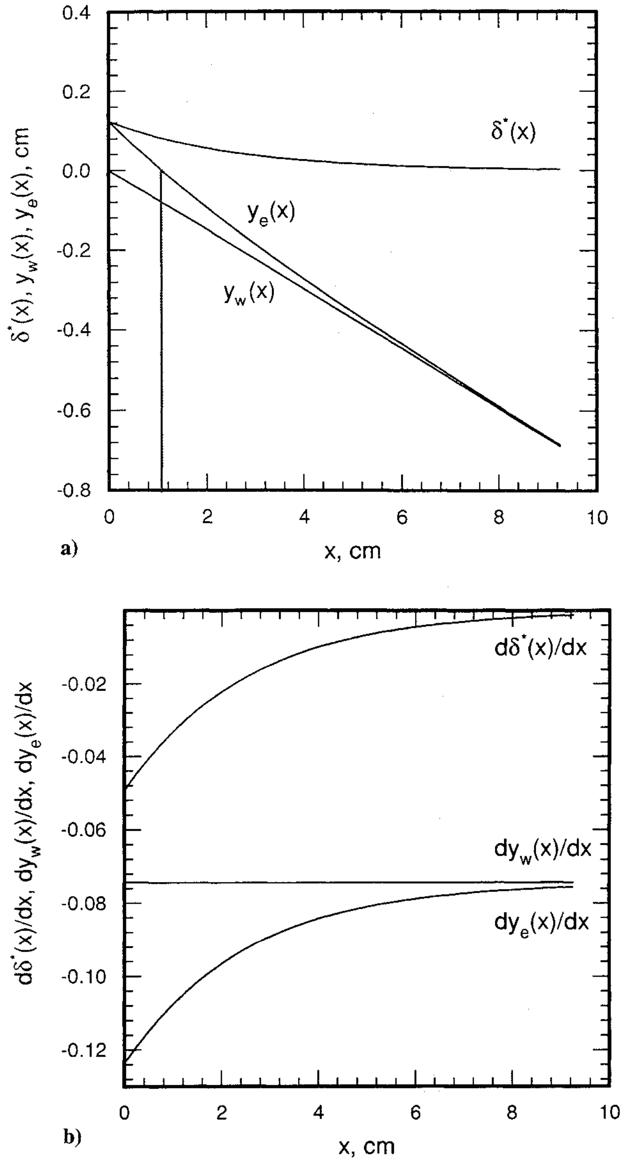

Fig. 5 The relative effects of displacement thickness and incidence for 4.25-deg expansion in a Mach 8 flow: a) geometric parameters and b) slope of geometric parameters.

final inviscid values of the pressure downstream of the expansion corners. Moreover, the theory shows good agreement with experiment in obtaining the downstream influence of the expansion.[9]

The pressure distribution of a 4.25-deg expansion corner obtained by the present method is compared with the experimental data given by Lu and Chung[9] in Fig. 3. The initial conditions for this case are identical to those of the 2.5-deg expansion corner presented earlier. The agreement between the theoretical and the experimental data is remarkably good. As expected the calculated pressure tends asymptotically to the inviscid two-dimensional value of the pressure farther downstream.

The effects of displacement thickness and incidence through modifying the geometry is now discussed. Plotted in Figs. 4a and 5a are the geometric body shape $y_w(x)$, the effective body shape $y_e(x)$, and the displacement thickness $\delta^*(x)$ downstream of the 2.5-deg and 4.25-deg expansion corners. Note that in both figures, the displacement thickness is given as the height above the wall in the y direction, i.e., above the zero ordinate. The effective body shape is then obtained from Eq. (7).

The following discussion will be based on the computed data for the 2.5-deg corner. The convex geometric body shape is merely a straight line with a negative slope of 2.5 deg. The displacement thickness and the effective body shape have the same value at the hinge line. Close to the hinge line over the ramp section where the effect of displacement thickness dominates the effect of incidence, the magnitude of the displacement thickness and the effective body shape are very close. As the flow moves downstream the effect of the displacement thickness is reduced, and the effect of incidence becomes more dominant. (In other words, $d\delta^*(x)/dx$ is approaching zero whereas $dy_w(x)/dx$ remains at a constant negative value, see Fig. 4b.) Therefore, the effective body shape is modified strongly by the displacement thickness near the front of the corner but becomes less affected downstream. Farther downstream, at $x = 5.45$ cm, the positive displacement thickness and the negative geometric body shape become equal; hence, $y_e(x) = 0$. This point, indicated as a vertical bar in Fig. 4a, can be viewed as the demarcation between the upstream region of significant viscous interaction and the downstream region where incidence predominates. The division of the interaction region in this manner is analogous to that for unseparated interactions at compression ramps which exhibit a sharp initial pressure rise.[5] Farther downstream, the displacement thickness continues to drop and the effective body shape becomes negative.

To reinforce the preceding observations, the rate of growth of the geometric body shape, the displacement thickness, and the effective body shape for the ramp section of the 2.5-deg expansion corner are compared in Fig. 4b. The rate of change of the displacement thickness and the geometric body shape are negative just after the hinge line. The sum of these two negative numbers results in a large negative value for the rate of change of the effective body shape in this region. Since the absolute rate of change of the effective body shape is large, the magnitude of the parameter K_v is also large. This large magnitude of K_v causes a very rapid drop of pressure in the region just downstream of the hinge line. (As mentioned in the preceding paragraph, the large initial pressure drop is analogous to the large initial pressure rise in unseparated interactions at compression ramps.) The rate of change of the displacement thickness increases moderately as the flow moves farther downstream which contributes to a slow increase of the rate of change of the effective body shape and K_v. This weak change of K_v downstream produces an asymptotic behavior of the pressure toward the end of the ramp.

The negative values obtained for the rate of change of the effective body shape suggest that K_v is also negative. Therefore, a pressure law such as Eq. (11) if used in conjunction with K_v will always produce a value of P greater than one. This is physically impossible beyond the hinge line of an expansion corner. The situation can be corrected by including a viscous correction term in this expression. This is in fact done in Eqs. (12) and (13).

For the stronger, 4.25-deg expansion, Fig. 5b shows that the absolute rate of change of displacement thickness is less than the ab-

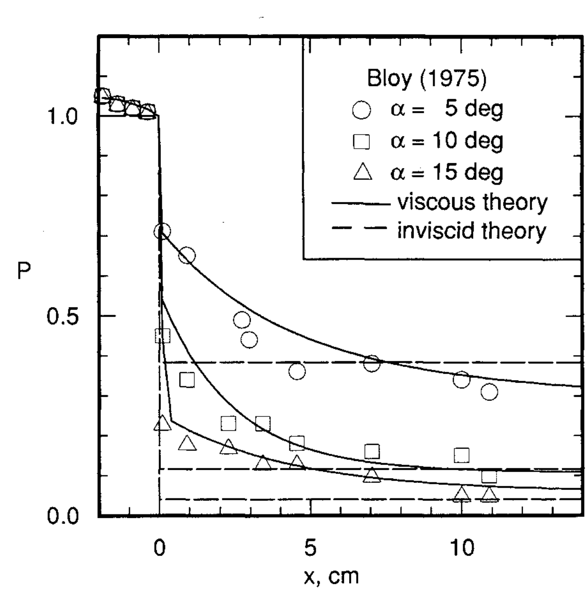

Fig. 6 Comparison with Bloy's data.[8]

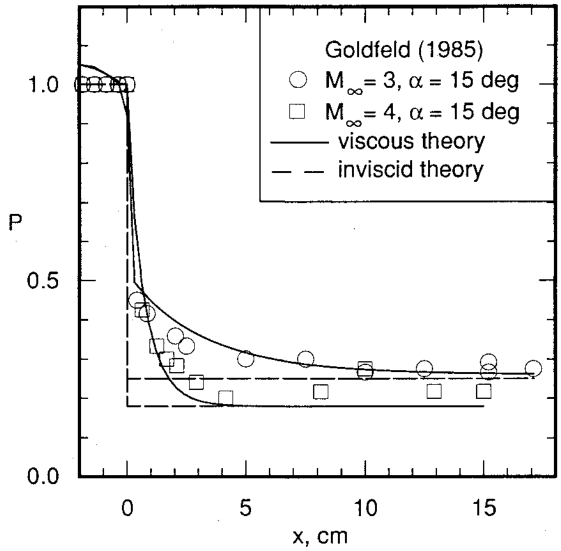

Fig. 7 Comparison with Goldfeld's data.[10]

solute value of the slope of the wall. Therefore, the effective body shape rapidly approaches the actual body shape. [In fact, $y_e(x) = 0$ at $x = 1.11$ cm as shown by a vertical bar in Fig. 5a.] This implies that viscous effects are confined close to the front of corner, giving rise to a large magnitude of K_v and a large pressure drop. A more gentle pressure decay is obtained farther downstream.

Next, Fig. 6 compares the present theoretical results for hypersonic, turbulent flow past 5-, 10-, and 15-deg expansion corners against Bloy's experimental data,[8] with the following conditions: $T_w = 280$ K, $T_0 = 2160$ K, $M_\infty = 7.35$, and $Re = 29 \times 10^6$/m. In the first case, Eq. (12) is used for the pressure law, whereas in the latter cases, Eq. (13) is used. Good agreement of the theory with experiment is achieved. The pressure can be seen to drop precipitously just downstream of the corner due to the rapid change of the boundary-layer displacement thickness. However, a large downstream influence is obtained due to the large value of K so that the pressure decays more gently toward the inviscid limit.[9]

Lu and Chung[9] observed that the pressure ratio across an inviscid Prandtl-Meyer expansion, which is an isentropic process, can be correlated by K throughout a wide range of supersonic and hypersonic Mach numbers. Moreover, the downstream influence also scales according to K. This behavior is also found in the present analysis when compared to supersonic flow,[10] see Fig. 7. (The conditions include a wall temperature ratio $T_w/T_0 = 1$ and $Re = 2.2 \times 10^6$/m.) The theory, using Eq. (12) for the Mach 3 case and Eq. (13) for the Mach 4 case, agrees well with experiment at both Mach numbers. Moreover, the asymptotic approach of the viscous results to the inviscid downstream pressure is also captured.

IV. Conclusions

The hypersonic, turbulent viscous interaction problem at an expansion corner was solved in a similar way to that described by Stollery and Bates.[5] The present analysis found that the flow is governed by viscous effects through a boundary-layer displacement on the wall contour and by incidence effects arising from the corner angle. The general form of a pressure law must take into account these effects. Based on an examination of experimental data, two pressure law approximations were proposed, one for weak expansion and the other for strong expansion. Solutions were obtained by assuming that the initially unknown pressure decays exponentially and by matching the pressure at the corner with that obtained upstream. Thus, no upstream influence was allowed. The study showed that the pressure decays gently for weak expansions but that it decays rapidly with a long asymptotic downstream region for strong expansions. This is because, for weak expansions, viscous effects through a large displacement thickness are relatively important. For strong expansions, the displacement thickness decreases rapidly to cause a rapid decay of pressure. Thus, viscous effects are important only near the front of the corner, and the surface pressure distribution appears more similar to the inviscid one. Finally, in a more practical sense, the approach outlined provides a quick way of estimating boundary-layer properties.

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